G.52.1 (numbered in 1766 by the Swedish mathematician Fredric Mallet in his Catalogue of Klingenstierna's Manuscripts)

Londini 5. Sept. 1729

Sit Ellipseos axis traversus $a$, distantia inter centrum & focorum alter utrum $c$. Dico excessum circumferentia circuli diametro $a$ descripti supra perimetrum Ellipseos esse ad illam circumferentiam ut est summam hoc seriei

$$\frac{cc}{aa} + \frac{1 \cdot 3 \cdot cc}{4aa} \times A + \frac{3 \cdot 5 \cdot cc}{9aa} \times B + \frac{5 \cdot 7 \cdot cc}{16aa} \times C + \frac{7 \cdot 9 \cdot cc}{25aa} \times D + \&\mathrm{c}$$

ad unitatem.

Coroll. Manente axe transverso $a$ augentur distantia foci $a$ centro $c$, donec fiat $c = \frac{1}{2}a$, & Ellipsis evanescens coincidet cum axe transverso seu diametro circuli circumscripti, & perimeter Ellipseos erit dupla hujus diametri. Hinc excessus circumferentia circuli supra duplum sua diametri est ad circumferentiam ut est summa seriei

$$\frac{1}{4} + \frac{1 \cdot 3}{16} \times A + \frac{3 \cdot 5}{36} \times B + \frac{5 \cdot 7}{64} \times C + \frac{7 \cdot 9}{100} \times D + \&\mathrm{c}$$

ad unitatem.

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The next section is a manuscript on another piece of paper. Mallet ordered it together with the manuscript above, but it could of course have been written later (or earlier).

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Ellipseos semiaxis transversus dicatur $e$, semiaxis conjugatus $b$, abscissa e centro $x$, & erit elementum arcus Elliptici

$$\frac{\sqrt{ee - bb}}{e} \times dx \sqrt{\frac{\frac{e^4}{ee-bb} - xx}{ee - xx}}$$

Numerator $\sqrt{\frac{e^4}{ee-bb} - xx}$ resolvatur in seriem, cujus singuli termini multiplicati per $\frac{dx}{e}\sqrt{\frac{ee-bb}{ee-xx}}$, & deinde ad rectificationem arcus circularis reducti, dant solutionem allatam, omnibus terminis algebraicis in casu $x = e$ evanescentibus.

Elem. arc. Ellipt: $\frac{c}{e} dx \sqrt{\frac{\frac{e^4}{cc} - xx}{ee - xx}}$